AMRIT SCIENCE CAMPUS

B.Sc. CSIT Second Semester

Pre-Semester Examination - **Set "B"**

Subject: Mathematics - II — Full Marks: 60

Time: 3 hours — Pass Marks: 24

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Group A $[2 \times 10 = 20]$**.**

Attempt ***any two*** *questions.*

1. Distinguish between a linear function $f : \mathbb{R} \longrightarrow \mathbb{R}$ and a linear transformation $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$. Prove that $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ defined by $T(x, y) = (4x - y, 2x + y)$ is a linear transformation. Let $S = \{\vec{e}_1, \vec{e}_2\}$ be the standard basis of Euclidean space $\mathbb{R}^2$. Under the given linear transformation T, what will be the area of the image of the parallelogram whose adjacent sides are given by the vectors in S? [2+3+5=10]

2. Define eigenvalues, eigenvectors, eigenspace, characteristic polynomial and characteristic equation for a square matrix. Find every possible eigenspace for $A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$. [2+8=10]

3. Define least-squares solution for an inconsistent system $A\vec{x} = \vec{b}$ and the error associated with this solution. Write the matrix equation which will give the normal equations for obtaining the least-squares solution. Obtain the least squares solution for the considered system if

$$A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}.$$

Also, obtain the least-squares error associated with this solution. [1.5+0.5+7+1=10]

Group B $[8 \times 5 = 40]$**.**

Attempt ***any eight*** *questions.*

4. Is it necessary that every system of linear equations is consistent? What kind of system of linear equations is always consistent? Given $A = \begin{bmatrix} 5 & 7 & 9 \\ 0 & 2 & 4 \\ 0 & -6 & -8 \end{bmatrix}$, is the system $A\vec{x} = \vec{b}$ consistent for all $\vec{b} \in \mathbb{R}^3$? Give a suitable explanation. [1+1+3=5]

5. If possible, find A^{-1} where $A = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 7 & -1 & 0 & 0 \\ 2 & 6 & 3 & 0 \\ 5 & 8 & 4 & -3 \end{bmatrix}$. [5]

6. Define subspace of a Euclidean space. Show that $H = \{(x, y, z) : x+y+z = 2x+3y+5z = 0\} \subseteq \mathbb{R}^3$ is a subspace of $\mathbb{R}^3$ and express H in terms of spanning vectors. [1+4=5]

7. Given $n \in \mathbb{N}$, define the polynomial space P_n over $\mathbb{R}$. What is the standard basis for this space? Find the standard coordinates of the polynomial represented by $p(x) = 9x+5$ in the space P_2. Find the change of coordinate matrix for P_2 which changes the B-coordinates of a given polynomial to standard coordinates, where $B = \{(x-1)^2, 4x^2 - 5x + 3, x(2x + 3)\}$ is the given non-standard basis. [1+0.5+0.5+3=5]

8. Define the associated matrix for the linear transformation $T : V \longrightarrow \mathcal{U}$ relative to the bases B (for V) and C (for $\mathcal{U}$), where V and $\mathcal{U}$ are vector spaces over the same field $\mathbb{K}$. Given that $T : P_3 \longrightarrow P_2$ is a linear transformation defined (formally) by $T(a_0+a_1x+a_2x^2+a_3x^3) = a_1+2a_2x+3a_3x^2$, find the associated matrix for T relative to the standard bases for P_3 and P_2. [1+4=5]

9. Obtain the general solution to the system of differential equations

$$\frac{dx}{dt} = -1.5x + 0.5y, \quad \frac{dy}{dt} = x - y$$

using the concept of eigenvalues. [5]

10. Define probability vector and stochastic matrix. Using these concepts, define Markov chain as a difference equation. It is known that RAMs installed in some computers of the CSIT department are not in a perfect condition and show some anomalous behavior. They either work efficiently or the computer will shut down due to the RAM getting heated. It is known that of all the RAMs that are working efficiently today, 95% will also work efficiently tomorrow. It is also known that of all the RAMs that got heated today, 55% will also get heated tomorrow. Using the concept of Markov chains, find the probability that the RAM working efficiently today will also work efficiently two days later. [1+0.5+3.5=5]

11. Define group. Prove that $\langle G, +\rangle$ is a group where $G = \{x + y\sqrt{2} : x, y \in \mathbb{Q}\}$ and $+$ indicates the usual addition of real numbers. [1+4=5]

12. Define field. Prove that $\mathbb{Z}_5$ is a field. [2.5+2.5=5]

BEST OF LUCK